TMDs and light-cone quark models

Sigfrido Boffi

Università di Pavia, Dipartimento di Fisica Nucleare e Teorica
Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Italy

in collaboration with Barbara Pasquini
+ Sofia Cazzaniga, Anatolij V. Efremov, Peter Schweitzer

Workshop on Partonic Transverse Momentum Distributions

Milos, September 29, 2009

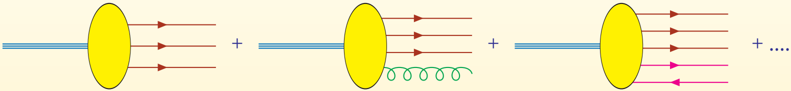
table of contents

References:

– B. Pasquini, S. Cazzaniga, S. B., Phys. Rev. D 78 (2008) 034025
– S. B., A. V. Efremov, B. Pasquini, P. Schweitzer, Phys. Rev. D 79 (2009) 094012
– B. Pasquini, M. Pincetti, S. B., Phys. Rev. D 72 (2005) 094029
– B. Pasquini, M. Pincetti, S. B., Phys. Rev. D 76 (2007) 034020
– B. Pasquini, S. B., Phys. Lett. B 653 (2007) 23
– S. B., B. Pasquini, Riv. Nuovo Cim. 30 (2007) 387, arXiv:0711.2625 [hep-ph]

A light-cone quark model for the nucleon

$$|N\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots$$

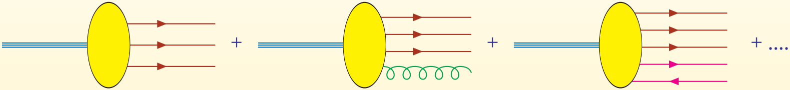


6 terms | 126 terms | many more terms

X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652 (2003) 383; Eur. Phys. J. C 33 (2004) 75

A light-cone quark model for the nucleon

$$|N\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots$$



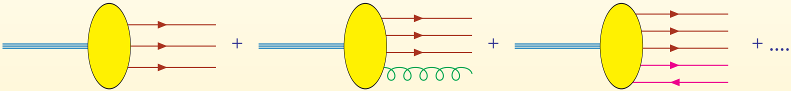
6 terms | 126 terms | many more terms

X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652 (2003) 383; Eur. Phys. J. C 33 (2004) 75

- basic assumptions

A light-cone quark model for the nucleon

$$|N\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots$$



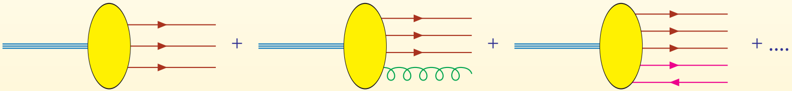
6 terms | 126 terms | many more terms

X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652 (2003) 383; Eur. Phys. J. C 33 (2004) 75

- basic assumptions

- only valence quarks

$$|N\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots$$



6 terms　　126 terms　　many more terms

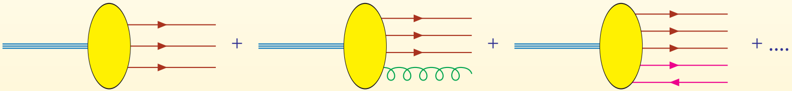
X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652 (2003) 383; Eur. Phys. J. C 33 (2004) 75

- basic assumptions

- only valence quarks
- described by SU(6) symmetric canonical w.f.: $\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})\,\Phi(\{\lambda_i\},\{\tau_i\})$

A light-cone quark model for the nucleon

$$|N\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \ldots$$



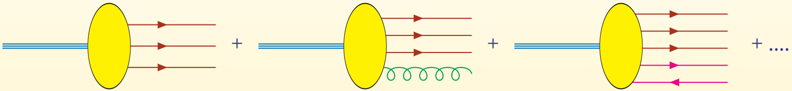
6 terms 126 terms many more terms

X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652 (2003) 383; Eur. Phys. J. C 33 (2004) 75

- basic assumptions

- only valence quarks
- described by SU(6) symmetric canonical w.f.: $\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})\,\Phi(\{\lambda_i\},\{\tau_i\})$
- light-cone w.f.s obtained by transforming from the instant to the light-front form

$$|N\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots$$



6 terms | 126 terms | many more terms

X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652 (2003) 383; Eur. Phys. J. C 33 (2004) 75

- basic assumptions

- only valence quarks
- described by SU(6) symmetric canonical w.f.: $\psi(\{x_i\}, \{\mathbf{k}_{i\perp}\})\,\Phi(\{\lambda_i\}, \{\tau_i\})$
- light-cone w.f.s obtained by transforming from the instant to the light-front form
- Melosh rotations introduce nonzero orbital angular momentum ($L_z = 0, \pm 1, 2$)

$$
\begin{aligned}
f_1^q(x,\mathbf{k}_T^2) &= N^q \int \mathrm{d}[X] |\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})|^2 \\
g_{1L}^q(x,\mathbf{k}_T^2) &= P^q \int \mathrm{d}[X] |\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})|^2 \frac{(m+xM_0)^2-\mathbf{k}_T^2}{(m+xM_0)^2+\mathbf{k}_T^2} \\
g_{1T}^q(x,\mathbf{k}_T^2) &= P^q \int \mathrm{d}[X] |\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})|^2 \frac{2M(m+xM_0)}{(m+xM_0)^2+\mathbf{k}_T^2} \\
h_1^q(x,\mathbf{k}_T^2) &= P^q \int \mathrm{d}[X] |\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})|^2 \frac{(m+xM_0)^2}{(m+xM_0)^2+\mathbf{k}_T^2} \\
h_{1T}^{\perp q}(x,\mathbf{k}_T^2) &= -P^q \int \mathrm{d}[X] |\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})|^2 \frac{2M^2}{(m+xM_0)^2+\mathbf{k}_T^2} \\
h_{1L}^{\perp q}(x,\mathbf{k}_T^2) &= -P^q \int \mathrm{d}[X] |\psi(\{x_i\},\{\mathbf{k}_{i\perp}\})|^2 \frac{2M(m+xM_0)}{(m+xM_0)^2+\mathbf{k}_T^2}
\end{aligned}
$$

$$
\mathrm{d}[X] = \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\,\delta\left(1-\sum_{i=1}^{3}x_i\right)\frac{\mathrm{d}^2\mathbf{k}_{1\perp}\,\mathrm{d}^2\mathbf{k}_{2\perp}\,\mathrm{d}^2\mathbf{k}_{3\perp}}{[2(2\pi^3)]^2}\,\delta\left(\sum_{i=1}^{3}\mathbf{k}_{i\perp}\right)\delta(x-x_3)\delta(\mathbf{k}_T-\mathbf{k}_{3\perp})
$$

as dictated by SU(6) symmetry: $N^u=2$, $N^d=1$, and $P^u=\frac{4}{3}$, $P^d=-\frac{1}{3}$

(model-dependent) relations between TMD

- in QCD all twist-2 and twist-3 TMDs are independent of each other
  $\Longrightarrow$ no exact relations among them
- model-dependent relations are possible

(model-dependent) relations between TMD

- in QCD all twist-2 and twist-3 TMDs are independent of each other
  $\Longrightarrow$ no exact relations among them
- model-dependent relations are possible
- in the light-cone quark model:

$$\begin{aligned} h_{1L}^{\perp q}(x, \mathbf{k}_\perp^2) &= -g_{1T}^q(x, \mathbf{k}_\perp^2) \\ \text{i.e. transv. pol. } q \text{ in long. pol. } N &= - \text{ long. pol. } q \text{ in transv. pol. } N \end{aligned}$$

- in QCD all twist-2 and twist-3 TMDs are independent of each other
  $\Longrightarrow$ no exact relations among them
- model-dependent relations are possible
- in the light-cone quark model:

$$\begin{aligned} h_{1L}^{\perp q}(x,\mathbf{k}_\perp^2) &= -g_{1T}^q(x,\mathbf{k}_\perp^2) \\ \text{i.e. transv. pol. } q \text{ in long. pol. } N &= - \text{ long. pol. } q \text{ in transv. pol. } N \end{aligned}$$

$$\begin{aligned} 2h_1^q(x,\mathbf{k}_\perp^2) &= g_{1L}^q(x,\mathbf{k}_\perp^2) + \frac{P^q}{N^q} f_1^q(x,\mathbf{k}_\perp^2) \qquad (*) \\ \frac{P^q}{N^q} f_1^q(x,\mathbf{k}_\perp^2) &= h_1^q(x,\mathbf{k}_\perp^2) - \frac{\mathbf{k}_\perp^2}{2M^2} h_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) \qquad (**) \end{aligned}$$

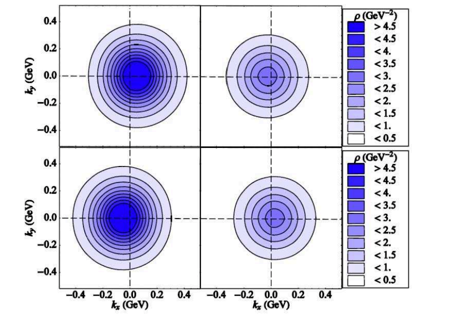
$(*) - (**) \Rightarrow$

$$g_{1L}^q(x,\mathbf{k}_\perp^2) - h_1^q(x,\mathbf{k}_\perp^2) = \frac{\mathbf{k}_\perp^2}{2M^2} h_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)$$

i.e. difference $g_1 - h_1$ measures relativistic effects encoded in $h_{1T}^{\perp q}$

$h_{1L}^{\perp} = -g_{1T}$

light-cone quark model



Phys. Rev. D 78 (2008) 034025

$$g_{1T} = -h_{1L}^{\perp}$$

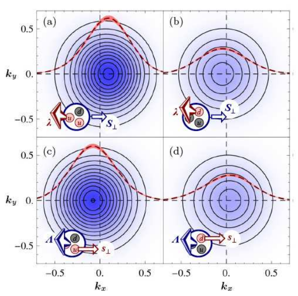
$\langle k_x \rangle = \;\; 55.81$ MeV (up)

$\langle k_x \rangle = -28.14$ MeV (down)

$\int dx\, g_{1T}$

$\int dx\, h_{1L}^{\perp}$

lattice calculation



Ph. Hägler *et al.*, arXiv:0908.1283 [hep-lat]

g_{1T} : $\langle k_x \rangle = \;\; 67(5)$ MeV

$h_{1L}^{\perp}$: $\langle k_x \rangle = -60(5)$ MeV (up)

g_{1T} : $\langle k_x \rangle = -30(5)$ MeV

$h_{1L}^{\perp}$: $\langle k_x \rangle = \;\; 16(5)$ MeV (down)

- light-cone quark model: $g_{1T} = -h_{1L}^{\perp}$
- lattice calculation: $g_{1T} \approx -h_{1L}^{\perp}$

$g_{1L} - h_1 = \frac{\mathbf{k}_\perp^2}{2M^2}\, h_{1T}^\perp$

$$\int d^2\mathbf{k}_\perp [g_{1L}(x,\mathbf{k}_\perp^2) - h_1(x,\mathbf{k}_\perp^2)] = \int d^2\mathbf{k}_\perp \frac{\mathbf{k}_\perp^2}{2M^2}\, h_{1T}^\perp(x,\mathbf{k}_\perp^2) \equiv h_{1T}^{\perp(1)}(x)$$

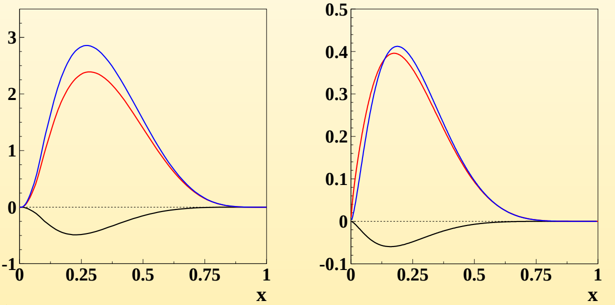
at the model scale | after evolution to $Q^2 = 2.5$ GeV^2

up quarks

—— $x\, g_1(x)$

—— $x\, h_1(x)$

—— $x\, h_{1T}^{\perp(1)}(x)$

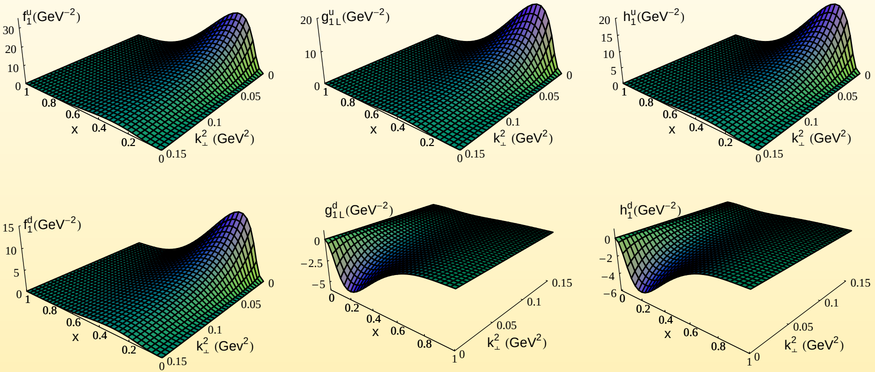


- “approximate” evolution of $h_{1T}^\perp$ using evolution equations of transversity

f_1, g_1, h_1

f_1 g_1 h_1

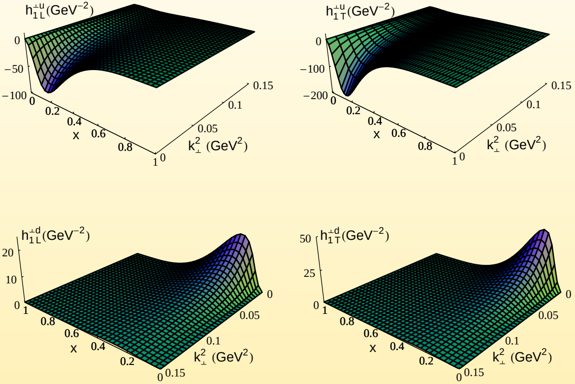
up



down

$h_{1L}^{\perp}$ $h_{1T}^{\perp}$

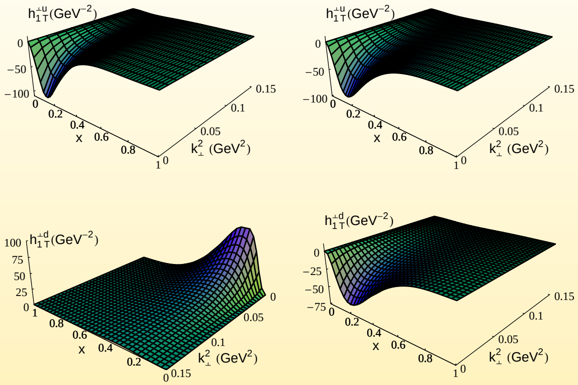
up



down

angular momentum decomposition of $h_{1T}^{\perp}$

up



down

$L_z = \pm 1$ $L_z = 0, 2$

orbital angular momentum content

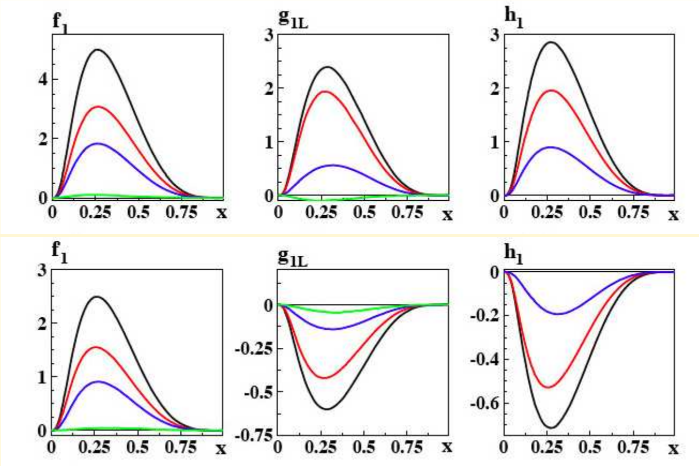
integrating over $\mathbf{k}_T$

up

- —— TOT
- —— S wave
- —— P wave
- —— D wave

down

- —— TOT
- —— S wave
- —— P wave
- —— D wave

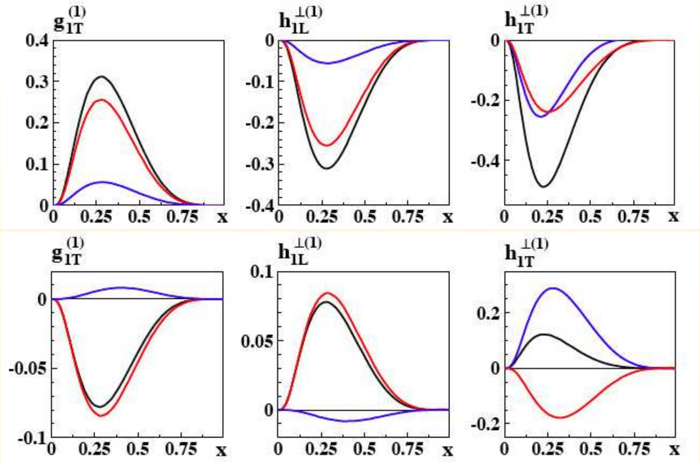


- total results obey SU(6) symmetry: $f_1^u = 2f_1^d$, $g_{1L}^u = -4g_{1L}^d$, $h_1^u = -4h_1^d$
- partial wave contributions do not!

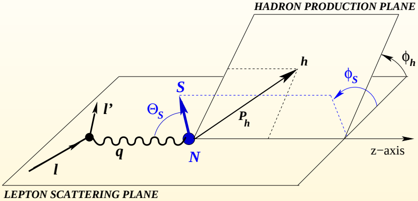
integrating over $\mathbf{k}_T$

up
—— TOT
—— S-P int.
—— P-D int.

down
—— TOT
—— S-P int.
—— P-D int.



- total results obey SU(6) symmetry: $g_{1T}^u = -4g_{1T}^d$, $h_{1L}^{\perp u} = -4h_{1L}^{\perp d}$, $h_{1T}^{\perp u} = -4h_{1T}^{\perp d}$
- partial wave contributions do not!



$$A_{XY}^{\text{weight}} = \frac{F_{XY}^{\text{weight}}}{F_{UU}}$$

$$X = \text{ beam pol.}$$
$$Y = \text{ target pol.}$$
$$\text{weight} = \text{ang. distr. hadron}$$

$$\frac{d^4\sigma}{dx\,dy\,dz\,d\phi_h} = \frac{d^4\sigma_0}{dx\,dy\,dz\,d\phi_h}\Big\{1 + \cos(2\phi_h)p_1(y)\,A_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)p_1(y)\,A_{UL}^{\sin(2\phi_h)}$$
$$+ \lambda S_L p_2(y)\,A_{LL} + \lambda S_T\cos(\phi_h-\phi_S)p_2(y)\,A_{LT}^{\cos(\phi_h-\phi_S)} + S_T\sin(\phi_h-\phi_S)A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+ S_T\sin(\phi_h+\phi_S)p_1(y)A_{UT}^{\sin(\phi_h+\phi_S)} + S_T\sin(3\phi_h-\phi_S)p_1(y)A_{UT}^{\sin(3\phi_h-\phi_S)}\Big\} + \dots$$

$$F_{UU} \sim f_1 \otimes D_1$$
$$F_{LL} \sim g_{1L} \otimes D_1$$
$$F_{UT}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$
$$F_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$$

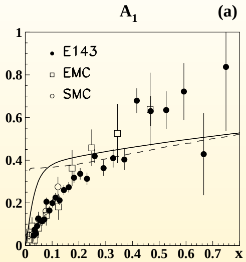
$$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$$
$$F_{UU}^{\cos(2\phi_h)} \sim h_1^{\perp} \otimes H_1^{\perp}$$
$$F_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$$
$$F_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp} \otimes H_1^{\perp}$$

- inclusive longitudinal asymmetry

no complications due to $k_\perp$ dependence
evolution equations known

$$A_1^p = \frac{\sum_a e_a^2\, x\, g_1^a(x)}{\sum_a e_a^2\, x\, f_1^a(x)}$$

SMC data at $\langle Q^2 \rangle = 3\ \text{GeV}^2$



a test under "controlled conditions": A_1, A_{LL}

- inclusive longitudinal asymmetry

no complications due to $k_\perp$ dependence
evolution equations known

$$A_1^p = \frac{\sum_a e_a^2 \, x \, g_1^a(x)}{\sum_a e_a^2 \, x \, f_1^a(x)}$$

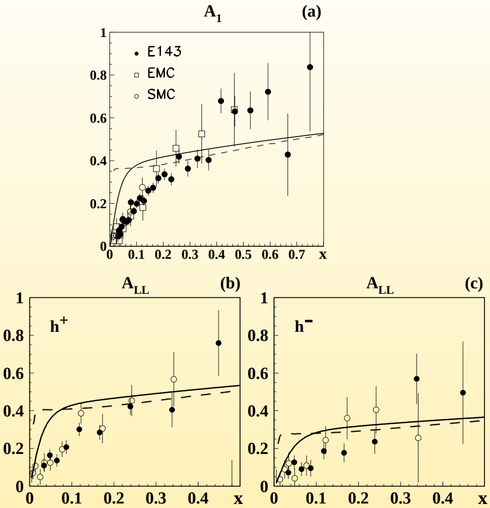
SMC data at $\langle Q^2 \rangle = 3$ GeV^2

- double-spin asymmetry

$$A_{LL} = \frac{\sum_a e_a^2 \, x \, g_1^a(x) \, D_1^a(z)}{\sum_a e_a^2 \, x \, f_1^a(x) \, D_1^a(z)}$$

$D_1(z)$ at $\langle Q^2 \rangle = 2.5$ GeV^2 from
S. Kretzer, PRD 62, 054001 (2000)

ϕ SMC ⧫ HERMES



—— $g_1(x), f_1(x)$ at initial scale – – – $g_1(x), f_1(x)$ evolved at $\langle Q^2 \rangle$

Gaussian ansatz: $f(x, k_\perp^2) = f(x)\,\exp[-k_\perp^2/\langle k_\perp^2\rangle]/\pi\langle k_\perp^2\rangle$

- $\mathbf{k}_T$-dependence not of Gaussian shape

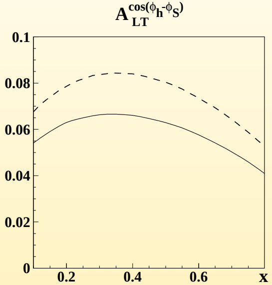
Gaussian ansatz: $f(x, k_\perp^2) = f(x)\ \exp[-k_\perp^2/\langle k_\perp^2\rangle]/\pi\langle k_\perp^2\rangle$

- $\mathbf{k}_T$-dependence not of Gaussian shape
- if transverse momenta were Gaussian, then the ratio in the fourth column would be unity

TMD	$\langle k_T\rangle$ (GeV)	$\langle k_T^2\rangle$ (GeV^2)	$\frac{4\langle k_T\rangle^2}{\pi\langle k_T^2\rangle}$
f_1	0.239	0.080	0.909
g_1	0.206	0.059	0.916
h_1	0.210	0.063	0.891
$g_{1T}^\perp$	0.206	0.059	0.916
$h_{1L}^\perp$	0.206	0.059	0.916
$h_{1T}^\perp$	0.190	0.050	0.919

- $\mathbf{k}_T$-dependence not of Gaussian shape
- if transverse momenta were Gaussian, then the ratio in the fourth column would be unity

TMD	$\langle k_T\rangle$ (GeV)	$\langle k_T^2\rangle$ (GeV^2)	$\frac{4\langle k_T\rangle^2}{\pi\langle k_T^2\rangle}$
f_1	0.239	0.080	0.909
g_1	0.206	0.059	0.916
h_1	0.210	0.063	0.891
$g_{1T}^\perp$	0.206	0.059	0.916
$h_{1L}^\perp$	0.206	0.059	0.916
$h_{1T}^\perp$	0.190	0.050	0.919



test:

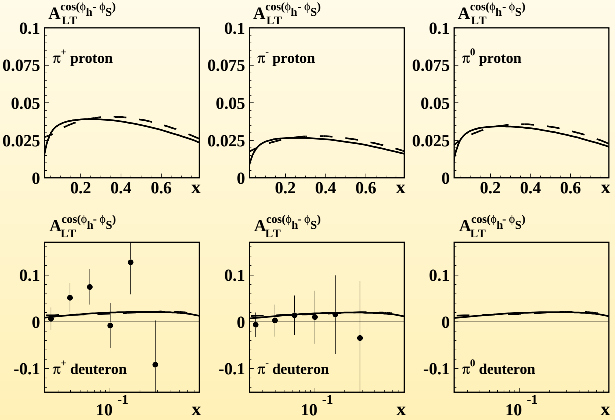
$$A_{LT}^{\cos(\phi_h-\phi_S)}(x) = \frac{\sum_a e_a^2\, x\, g_{1T}^{(1)a}(x)\, D_1^a}{\sum_a e_a^2\, x\, f_1^a(x)\, D_1^a}$$

– – – with Gaussian ansatz

——— exact result

$D_1(z)$ from Bacchetta *el.* PLB 659, 234 (2008)

$$A_{LT}^{\cos(\phi_h-\phi_S)}(x) = \frac{\sum_a e_a^2\, x\, g_{1T}^{(1)a}(x)\, \langle D_1^a \rangle}{\sum_a e_a^2\, x\, f_1^a(x)\, \langle D_1^a \rangle}$$



– – – at the initial scale —— at 2.5 GeV^2 COMPASS preliminary data

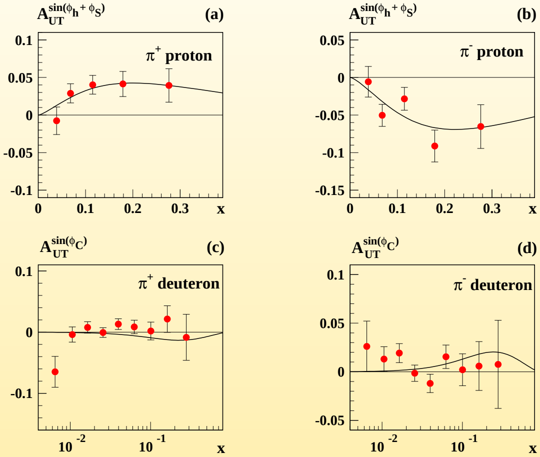
$$A_{UT}^{\sin(\phi_h+\phi_S)}(x) = \frac{\sum_a e_a^2\, x\, h_1^a(x)\, \langle H_1^{\perp(1/2)a} \rangle}{\sum_a e_a^2\, x\, f_1^a(x)\, \langle D_1^a \rangle}$$

$H_1^{\perp(1/2)a}$ from A.V. Efremov *et al.*, PRD 73, 094025 (2006)

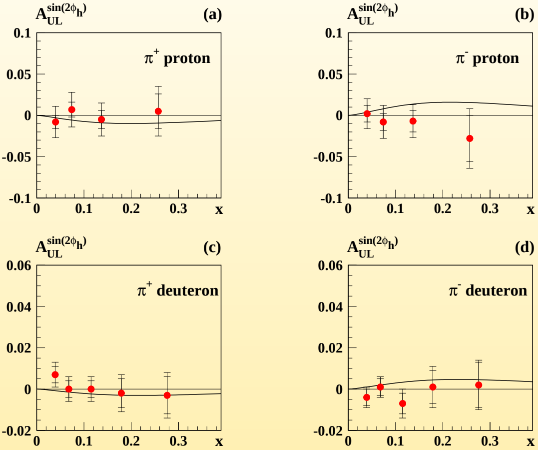
HERMES data
EPJ A 38, 145 (2008)

COMPASS data
PLB 673, 127 (2009)

$$\phi_C = \phi_h + \phi_S + \pi$$
$$A_{UT}^{\sin(\phi_C)} = -A_{UT}^{\sin(\phi_h+\phi_S)}$$



$$A_{UL}^{\sin 2(\phi_h)}(x) = \frac{\sum_a e_a^2\, x\, h_{1L}^{\perp(1)a}(x)\, \langle H_1^{\perp(1/2)a}\rangle}{\sum_a e_a^2\, x\, f_1^a(x)\, \langle D_1^a\rangle}$$



HERMES data
PRL 84, 4047 (2000)
NP B Suppl. 79, 523 (1999)

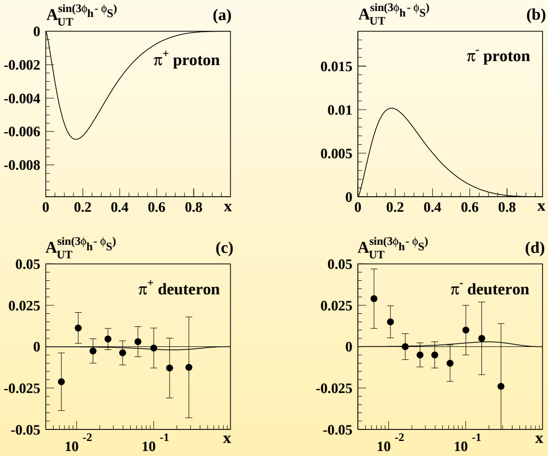
$A_{UT}^{\sin(3\phi_h-\phi_s)}$

$$A_{UT}^{\sin(3\phi_h-\phi_s)}(x) = -\frac{\sum_a e_a^2\, x\, h_{1T}^{\perp(1)a}(x)\, \langle H_1^{\perp(1/2)a}\rangle}{\sum_a e_a^2\, x\, f_1^a(x)\, \langle D_1^a\rangle}$$

experiment planned
at CLAS12
H. Avakian *et al.*, LOI 12-06-108

HERMES data analysis
in progress

COMPASS data
arXiv:0705.2402



conclusions

- time-even TMDs calculated in a light-cone quark model

- time-even TMDs calculated in a light-cone quark model
- three (model-dependent) relations found among TMDs

conclusions

- time-even TMDs calculated in a light-cone quark model
- three (model-dependent) relations found among TMDs
- TMDs not of Gaussian shape in $\mathbf{k}_T$, but Gauss approximation reasonable

- time-even TMDs calculated in a light-cone quark model
- three (model-dependent) relations found among TMDs
- TMDs not of Gaussian shape in $\mathbf{k}_T$, but Gauss approximation reasonable
- predictions presented for all the leading-twist azimuthal spin asymmetries in SIDIS due to time-even TMDs

conclusions

- time-even TMDs calculated in a light-cone quark model
- three (model-dependent) relations found among TMDs
- TMDs not of Gaussian shape in $\mathbf{k}_T$, but Gauss approximation reasonable
- predictions presented for all the leading-twist azimuthal spin asymmetries in SIDIS due to time-even TMDs
- results in fair agreement with data (Collins SSA the only one not compatible with zero)

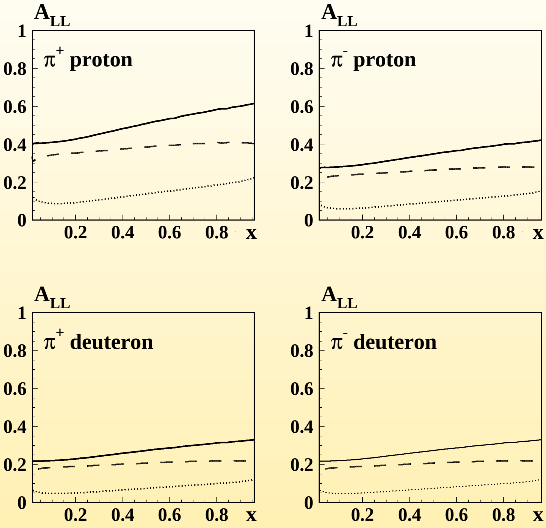
- time-even TMDs calculated in a light-cone quark model
- three (model-dependent) relations found among TMDs
- TMDs not of Gaussian shape in $\mathbf{k}_T$, but Gauss approximation reasonable
- predictions presented for all the leading-twist azimuthal spin asymmetries in SIDIS due to time-even TMDs
- results in fair agreement with data (Collins SSA the only one not compatible with zero)

- Thank you for your attention

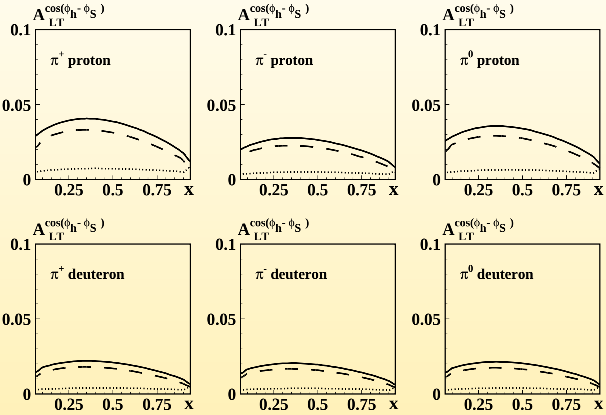
angular momentum decomposition: A_{LL}

f_1 and g_1 at the model scale

——— total
– – – S wave
· · · · · P wave



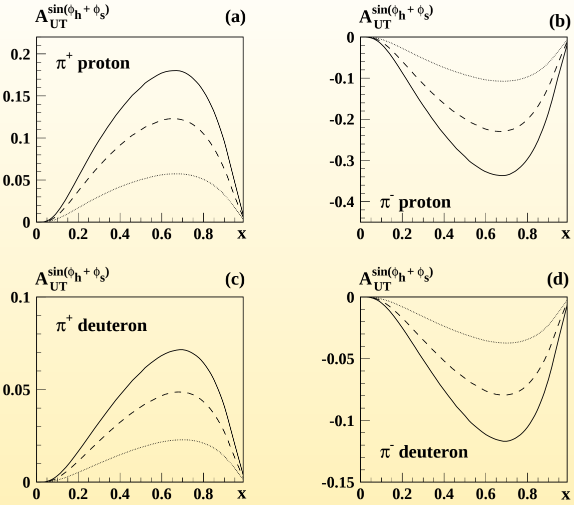
f_1 and $g_{1T}^{(1)}$ at the model scale



——— total   – – – S-P interference   · · · · · P-D interference

h_1 at the model scale
f_1 from GRV at 2.5 GeV^2

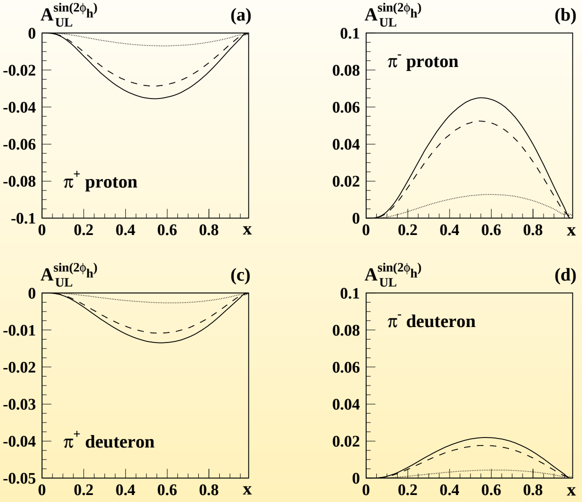
——— total
– – – S wave
· · · · · P wave



angular momentum decomposition: $A_{UL}^{\sin(2\phi_h)}$

$h_{1L}^{\perp(1)}$ at the model scale
f_1 from GRV at 2.5 GeV^2

——— total
– – – S-P interference
· · · · · P-D interference



angular momentum decomposition: $A_{UT}^{\sin(3\phi_h-\phi_S)}$

$h_{1T}^{\perp(1)}$ at the model scale
f_1 from GRV at 2.5 GeV^2

——— total
$- - -$ $L_z = \pm 1$ interference
$\cdots\cdots$ $L_z = 0, 2$ interference